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A PRELIMINARY STUDY OF NUMERICAL SOLUTIONS FOR FULLY FUZZY LINEAR SYSTEM

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Abstract

Fuzzy linear system (FLS) is differs from conventional linear system which is the coefficients in the linear equation are fuzzy. Fully fuzzy linear system (FFLS) is one of the important kinds of FLS which all the parameters are fuzzy numbers. This study begins with description on basic concept of Fuzzy Theory, FLS, and FFLS. This study are discussed the decomposition and extended decomposition methods in order to solve different type of FFLS problems. Decomposition method is a method to solve FFLS problems that not containing constraints whereas extended decomposition method is applicable to solve FFLS problems that contain constraints. By making use of the MATLAB programming, MAPLE programming, and LINGO solver of both methods, it speeded up the time of computation of simplex and Doolittle LU methods. The objective of the research is to compare the method of FFLS between decomposition and extended decomposition methods of FFLS equation. Three examples of FFLS problem have been chosen in this study. It is interesting that the solution obtained in Example 1 and Example 2 usually same for both methods and can be solved successfully meanwhile Example 3 only can be solved by extended decomposition method. This results show that extended decomposition method performs better than decomposition method.

Keywords: Decomposition Method; Doolittle LU Method; Extended Decomposition Method; Fully Fuzzy Linear System (FFLS); Simplex Method

1.0 INTRODUCTION

Linear system is a mathematical model of a system based on the use of a linear operator [1]. Linear system typically exhibit features and properties that are much simpler than the general, nonlinear case. As a mathematical idealization, linear systems find and play important applications in signal processing, control theory, robotic, wave propagations, electrical circuit and telecommunications [2]. Linear system can be represented in the matrix form of $A\ x = b$. A is an n x n matrix, x is a column vector with n entries, and b is a column vector with m entries.

Fuzzy linear systems (FLS) arise in many branches of sciences and technology such as economics, social sciences, telecommunications, image processing etc. FLS is differs from conventional linear system which is the coefficients in the linear equation are fuzzy. In the matrix form of $A\ x = b$, the matrix $A = (a_{ij})$, $1 \le i, j \le n$ is a crisp n x n matrix and $b_i \in E^1$, $1 \le i \le n$ [3].

System of equations is the simplest and the most useful mathematical model for many problems considered by applied mathematics. In practice, the exact values of coefficients of these systems are not a rule known. Fully Fuzzy Linear System (FFLS) can be described as mathematical method and one field of applied mathematics that has many functions in various area of solving a system of linear equations. FFLS differs from FLS in all the parameters of FFLS are fuzzy numbers [4]. In this study, system of linear equations $A\ x = b$ is considered where the element a_{ij} of the matrix A and the elements b_i of b are fuzzy numbers.

In this study, there are only two methods of FFLS to be considered, that is decomposition and extended decomposition method. The main objective of this study is to compare the method of FFLS between decomposition and extended decomposition methods of m x n FFLS equation.

2.0 LITERATURE REVIEW

A linear system is a mathematical model of a system based on the use of a linear operator and a collection of linear equations involving the same set of variables. Let A be a nonsingular matrix, so that A^{-1} is exact and unique solution. We consider the linear system as

A x = b

where exact solution is x = A^{-1} b.

Linear system typically exhibit properties that are much simpler than nonlinear case. A system of equations that has no solution is called inconsistent, where a system that has at least one solution is said to be consistent.

The matrix of coefficients A =

$$\begin{bmatrix} a_{11} & & a_{12} & \cdots & a_{1n} \\ a_{21} & \vdots & a_{22} & \ddots & a_{2n} \\ a_{m1} & & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

The augmented matrix $A^{\#}$ =

$$\begin{bmatrix} a_{11} & & a_{12} & & a_{1n} & & b_1 \\ a_{21} & \vdots & a_{22} & & a_{2n} & \vdots & b_2 \\ a_{m1} & & a_{m2} & \cdots & a_{mn} & & b_m \end{bmatrix}$$

The augmented matrix completely characterizes a system of equations since it contains the entire system coefficient and system constant. A system of equations is set of equation that deals with all together at once. It is much easier and simpler than nonlinear equations. The simplest system of linear equations is which contain two variables and two equations in one condition. The general system of m linear equations with n unknowns may be written in the form of

$$\begin{matrix} a_{11}x_1 & + & a_{12}x_2 & +\cdots+ & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & +\cdots+ & a_{2n}x_n & = & b_2 \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{m1}x_1 & + & a_{m2}x_2 & +\cdots+ & a_{mn}x_n & = & b_m \end{matrix}$$

where the system coefficient a_{mn} and the system constants b_n are given $x_1, x_2, \ldots, x_n$ denote by the unknown in system. The b_m = 0 for all m then the system is call homogenous, otherwise it is called non homogenous when $b_m \neq 0$.

- Plane have no intersection point
- Plane intersect in just one point
- Plane intersect in a line
- Plane that are coincident

There are three possibilities for the solution of system of three equations in three unknowns. The system either has no solution, it just unique solution, or it is has an infinite number of solutions. If the system is consistent then we have either unique solution or infinitely many solutions of the non-homogeneous system.

The system has a unique solution if and only if determinant of A is non zero (|A| ≠ 0) and the solution of the system may be written as $x = A^{-1}b$. The homogeneous system possesses only a trivial solution $x_1, x_2, \ldots, x_n$ = 0 if |A| ≠ 0. But it is interesting to note that a homogeneous system has also nontrivial solution provided |A| = 0 i.e. coefficient matrix is singular.

Systems of simultaneous linear equations play a major role in various areas such as mathematics, physics, statistics, engineering and social sciences. Since in many applications at least some of the system's parameters and measurements are represented by fuzzy rather than crisp number, it is immensely important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems and solve them. The concept of fuzzy number and arithmetic operations with these numbers in FLS were first introduced and investigated by L.A. Zadeh.

For constructing a system of linear equations it is assumed that there is no uncertainty about any parameter of the system but in real life there may exist uncertainty about some or all parameters. In such a case it is better to represent all the parameters of system of linear equations by fuzzy sets.

A particular type of FLS, $Ax = b$, where the coefficient matrix $A = (a_{ij})$ is in condition of fuzzy and $b = (y_i)$ is an arbitrary fuzzy number vector. Let we consider the n x n linear system

$$\begin{matrix} a_{11}x_1 & + & a_{12}x_2 & + \ldots + & a_{1n}x_n & = & y_{1,} \\ a_{21}x_1 & + & a_{22}x_2 & + \ldots + & a_{2n}x_n & = & y_{2,} \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{n1}x_1 & + & a_{n2}x_2 & + \ldots + & a_{nn}x_n & = & y_n, \end{matrix}$$

where the coefficient matrix A = (a_{ij}), 1 ≤ i, j ≤ n is a crisp n x n matrix and $y_i \in E^1$, 1 ≤ i ≤ n is called a FLS.

Let say we want to know about fuzzy numbers in FLS, so we represent an arbitrary fuzzy numbers by an ordered pair of functions $(\underset{\sim}{u}(r), \bar{u}(r)), 0 \leq r \leq 1$ which satisfy the following requirements:

1. $\underset{\sim}{u}$ (r) is a bounded left continuous nondecreasing function over [0,1].

2. ū (r) is a bounded left continuous nonincreasing function over [0,1].
3. ṵ (r) ≤ ū (r), 0 ≤ r ≤ 1.

For example, the fuzzy number (1+r, 4-2r) is shown in Figure 6. A crisp number a is simply represented by ṵ (r) = ū (r) = a, 0 ≤ r ≤ 1. By appropriate definitions the fuzzy number space {ṵ (r), ū (r)} becomes a convex cone E^1 which is then embedded isomorphically and isometrically into a Banach space.

Fully Fuzzy Linear System (FFLS) is one of the important kind of fuzzy linear systems in which all the parameters are fuzzy numbers. Let say we consider the m x n fully fuzzy linear system of equations:

$$\begin{array}{ccccc} \tilde{a}_{11}\times\tilde{x}_1 & +\cdots+ & \tilde{a}_{1n}\times\tilde{x}_n & = & \tilde{a}_1, \\ \tilde{a}_{21}\times\tilde{x}_1 & +\cdots+ & \tilde{a}_{2n}\times\tilde{x}_n & = & \tilde{a}_2, \\ \vdots & & \vdots & & \vdots \\ \tilde{a}_{m1}\times\tilde{x}_1 & +\cdots+ & \tilde{a}_{mn}\times\tilde{x}_n & = & \tilde{a}_m. \end{array}$$

The matrix form of the above equation is

$$\tilde{A}\ x\ \tilde{X} = \tilde{b},$$

where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, 1 ≤ i, j ≤ n is a n x n fuzzy matrix and $\tilde{a}_j, \tilde{b}_j \in F(R)$. This system is called a fully fuzzy linear system (FFLS). While for the positive solution of FFLS $\tilde{A}\ x\ \tilde{X} = \tilde{b}$, where $\tilde{A}$ = (A, M, N) > Õ, $\tilde{b}$= (b, g, h) > Õ, and $\tilde{X}$ = (x, y, z) > Õ. So we have:

(A, M, N) x (x, y, z) = (b, g, h).

Now if we assume that A is a nonsingular crisp matrix, then we can write:

(Ax, Ay + Mx, Az + Nx) = (b, g, h) →

$$\begin{cases} Ax = b, \\ Ay + Mx = g, \\ Az + Nx = h. \end{cases}$$

So,

$$\begin{cases} Ax = b \ \rightarrow \ x = A\ inverse\ b, \\ Ay = g - Mx \ \rightarrow \ y = A\ inverse\ g - A\ inverse\ Mx, \\ Az = h - Nx \ \rightarrow \ z = A\ inverse\ h - A\ inverse\ Nx. \end{cases}$$

FFLS theory is beneficial in various fields. There are some examples that applied FFLS theory.

3.0 DECOMPOSITION METHOD

Decomposition method is the modified of LU decomposition method. This decomposition method is suitable to solve FFLS problem that not contain constraint in smaller computing process. This method also performs better than LU decomposition method because it is easy and less time consuming to apply this decomposition method. We consider fully fuzzy linear system of equations as $A \otimes x = b$ where $A = (A,M,N)$, $x = (x,y,z)$, $b = (b,h,g)$, that is (A,M,N) x $(x,y,z) = (b,h,g)$. Let A be an n x n matrix with all non-zero leading principal minors. Then A has a unique factorization $A = L \otimes U$ where L is unit lower triangular and U is upper triangular. Assume that $A = (A,M,N)$, where A is a full rank crisp matrix. Then, we let $L = (L, 0, 0)$ and $L = (U_1, U_2, U_3)$. Here, we consider $L = (L, 0, 0)$ instead of $L = (L_1, L_2, L_3)$ because our aim is to find a lower triangular fuzzy matrix L with the diagonal of fuzzy identify number in LR fuzzy multiplication which is $I = (1,0,0)$, not $I = (1,1,1)$. So, $A = L \otimes U$ now becomes $(A,M,N) = (L_1,0,0) \otimes (U_1,U_2,U_3) \rightarrow (A,M,N) = (L_1U_1, L_1U_2, L_1U_3)$. Then we have $(L_1U_1, L_1U_2, L_1U_3) \otimes (x,,) = (b,h,g)$.

Therefore, by using scalar multiplication

$$\gamma \otimes (m,\alpha,\beta) = \begin{cases} (\gamma m, \gamma\alpha, \gamma\beta), \gamma \geq 0, \\ (\gamma m, -\gamma\beta, -\gamma\alpha), \gamma < 0. \end{cases} \tag{1}$$

Then we have

$$\begin{aligned} &(L_1U_1x, L_1U_2x + L_1U_1y, L_1U_3x + L_1U_1z) = (b,h,g) \\ &L_1U_1 = b, \\ &L_1U_2x + L_1U_1y = h, \\ &L_1U_3x + L_1U_1z = g, \end{aligned} \tag{2}$$

And therefore

$$\begin{aligned} &x = U_1^{-1}L_1^{-1}b, \\ &y = U_1^{-1}L_1^{-1}(h - L_1U_2x), \\ &z = U_1^{-1}L_1^{-1}(g - L_1U_3x). \end{aligned} \tag{3}$$

4.0 EXTENDED DECOMPOSITION METHOD

Extended decomposition method is a new method based upon the decomposition of an FFLS into a nonlinear system and subsequently a linear programming problem, is proposed to find the solution of FFLS without any restriction on coefficient and fuzzy variables. This method can solve the FFLSs problem which cannot be solved by any of the available methods due to the restrictions imposed by the methods. Hence the FFLS problems that have been solved belong to the following category which is a square FFLS with unrestricted fuzzy variables. In this study, extended decomposition method is presented to solve a fully fuzzy linear system with no restrictions on the parameters. The steps of the extended decomposition method starting with

Substitute $\tilde{A}=\left(\tilde{a}_{ij}\right)_{m\times n}$, $\tilde{x}=\left(\tilde{x}_j\right)_{n\times l}$, and $\tilde{B}=\left(\tilde{b}_j\right)_{m\times l}$ written as $\sum^{+}_{j=1,..,n}\tilde{a}_{ij}\otimes\tilde{x}_j=\tilde{B}_j, \forall i=1,2...,m.$

If all the parameter $\tilde{a}_{ij}, \tilde{x}_j$ and $\tilde{b}_j$ are represented by triangular fuzzy numbers (a_{ij},b_{ij},c_{ij}), (x_j,y_j,z_j), and (B_i,G_i,H_i), respectively, then the FFLS may be written as $\sum^{+}_{j=1,..,n}(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)=(B_i,G_i,H_i),\ \forall i=1,2,...,m.$

.Then we are assuming $(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)=(f_{ij},p_{ij},q_{ij})$, the FFLS, obtained in Step 2, may be written as $\sum^{+}_{j=1,..,n}(f_{ij},p_{ij},q_{ij})=(B_i,G_i,H_i), \forall i=1,2...,m$, where $(f_{ij},p_{ij},q_{ij})=(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)$. As $\tilde{a}_{ij}$ and $\tilde{x}_j$ are both arbitrary fuzzy numbers, the above product can be defined in three subcases.

Case 1. If $(a_{ij},b_{ij},c_{ij})\geq 0$, that is, $a_{ij}\geq 0$,

$$(f_{ij},p_{ij},q_{ij})=\left(\begin{array}{l}\left(\frac{a_{ij}+c_{ij}}{2}\right)x_j-\left(\frac{c_{ij}-a_{ij}}{2}\right)|x_j|, b_{ij}y_j,\\ \left(\frac{a_{ij}+c_{ij}}{2}\right)z_j+\left(\frac{c_{ij}-a_{ij}}{2}\right)|z_j|\end{array}\right). \quad (4)$$

Using Definition 7, the above equation may be written as,

$$\begin{aligned} f_{ij}&=\left(\frac{a_{ij}+c_{ij}}{2}\right)x_j-\left(\frac{c_{ij}-a_{ij}}{2}\right)|x_j|,\\ p_{ij}&=b_{ij}y_j,\\ q_{ij}&=\left(\frac{a_{ij}+c_{ij}}{2}\right)z_j+\left(\frac{c_{ij}-a_{ij}}{2}\right)|z_j|.\end{aligned} \quad (5)$$

To solve this system, we define variables x'_j and x''_j as

$$\begin{aligned} x_j' &=\begin{cases} x_j & if\ x_j>0,\\ 0 & otherwise\end{cases}\\ x_j'' &=\begin{cases} -x_j & if\ x_j<0,\\ 0 & otherwise\end{cases}\end{aligned} \quad (6)$$

Hence, we substitute $x_j=x'_j-x''_j$ and $|x_j|=x'_j+x''_j$; similarly define variables z'_j and z''_j; substitute $z_j=z'_j-z''_j$ and $|z_j|=z'_j+z''_j$, where $x'_j, x''_j, z'_j, z''_j\geq 0$. Therefore, we obtain the following linear system of equation:

$$\begin{aligned} f_{ij}&=\left(\frac{a_{ij}+c_{ij}}{2}\right)(x'_j-x''_j)-\left(\frac{c_{ij}-a_{ij}}{2}\right)(x'_j+x''_j),\\ p_{ij}&=b_{ij}y_j,\\ q_{ij}&=\left(\frac{a_{ij}+c_{ij}}{2}\right)(z'_j-z''_j)+\left(\frac{c_{ij}-a_{ij}}{2}\right)(z'_j+z''_j).\end{aligned} \quad (7)$$

Case 2. If $(a_{ij},b_{ij},c_{ij})\leq 0$, that is, $c_{ij}\leq 0$,

$$(f_{ij},p_{ij},q_{ij})=\left(\begin{array}{l}\left(\frac{a_{ij}+c_{ij}}{2}\right)z_j+\left(\frac{c_{ij}-a_{ij}}{2}\right)|z_j|, b_{ij}y_j,\\ \left(\frac{a_{ij}+c_{ij}}{2}\right)x_j-\left(\frac{c_{ij}-a_{ij}}{2}\right)|x_j|\end{array}\right). \quad (8)$$

This can be similarly reduced as follows. We are putting $x_j=x'_j-x''_j$ and $|x_j|=x'_j+x''_j$,
$z_j=z'_j-z''_j$ and $|z_j|=z'_j+z''_j$,
where $x'_j, x''_j, z'_j, z''_j\geq 0$. Thus

$$\begin{aligned} f_{ij}&=\left(\frac{a_{ij}+c_{ij}}{2}\right)(z'_j-z''_j)+\left(\frac{c_{ij}-a_{ij}}{2}\right)(z'_j+z''_j),\\ p_{ij}&=b_{ij}y_j,\\ q_{ij}&=\left(\frac{a_{ij}+c_{ij}}{2}\right)(x'_j-x''_j)-\left(\frac{c_{ij}-a_{ij}}{2}\right)(x'_j+x''_j).\end{aligned} \quad (9)$$

Case 3. If $(a_{ij},b_{ij},c_{ij})\approx 0$, that is, $a_{ij}<0<c_{ij}$,

$$(f_{ij},p_{ij},q_{ij})=\left(\begin{array}{l}\left(\frac{a_{ij}z_j+c_{ij}x_j}{2}\right)-\left|\frac{c_{ij}x_j-a_{ij}z_j}{2}\right|, b_{ij}y_j,\\ \left(\frac{a_{ij}x_j+c_{ij}z_j}{2}\right)+\left|\frac{c_{ij}z_j-a_{ij}x_j}{2}\right|\end{array}\right). \quad (10)$$

Using Definition 5, the above equation may be written as

$$\begin{aligned} f_{ij}&=\left(\frac{a_{ij}z_j+c_{ij}x_j}{2}\right)-\left|\frac{c_{ij}x_j-a_{ij}z_j}{2}\right|,\\ p_{ij}&=b_{ij}y_j,\\ q_{ij}&=\left(\frac{a_{ij}x_j+c_{ij}z_j}{2}\right)+\left|\frac{c_{ij}z_j-a_{ij}x_j}{2}\right|.\end{aligned} \quad (11)$$

This can be further reduced as follows. We are putting $x_j = x'_j - x''_j$ and $|x_j| = x'_j + x''_j$,
$z_j = z'_j - z''_j$ and $|z_j| = z'_j + z''_j$,
$c_{ij}x_j - a_{ij}z_j = u'_{ij} - u''_{ij}$ and $|c_{ij}x_j - a_{ij}z_j| = u'_{ij} + u''_{ij}$, $c_{ij}z_j - a_{ij}x_j = v'_{ij} - v''_{ij}$ and
$|c_{ij}z_j - a_{ij}x_j| = v'_{ij} + v''_{ij}$, where
$x'_j, x''_j, z'_j, z''_j, u'_{ij}, u''_{ij}, v'_{ij}, v''_{ij} \geq 0$.

On simplifying we obtain

$$f_{ij} = \left(\frac{a_{ij}(z'_j - z''_j) + c_{ij}(x'_j - x'_j)}{2}\right) - \left(\frac{u'_{ij} + u''}{2}\right),$$
$$p_{ij} = b_{ij}y_j, \tag{12}$$
$$q_{ij} = \left(\frac{a_{ij}(x'_j - x''_j) + c_{ij}(z'_j - z''_j)}{2}\right) - \left(\frac{v'_{ij} + v''_{ij}}{2}\right).$$

After having broken down the FFLS a set of linear equations, the system may be written as $\left(\sum_{j=1}^{n} f_{ij}, \sum_{j=1}^{n} p_{ij}, \sum_{j=1}^{n} f_{ij}\right) = (B_i, G_i, H_i), \forall i = 1,2..., m$ or equivalent with

$$\sum_{j=1}^{n} f_{ij} = B_i; i = 1,2..., m, \sum_{j=1}^{n} p_{ij} = G_i; i = 1,2..., m,$$
$$\sum_{j=1}^{n} f_{ij} = H_i; i = 1,2..., m$$

In order to solve the system, we decompose the FFLS into an equivalent linear programming problem LPP. For notational convenience we construct the following step functions:

$$\varepsilon_1\left(\tilde{A} = (a,b,c)\right) = \begin{cases} 1 & if\ \tilde{A} \geq 0, that is, a \geq 0, \\ 0 & otherwise \end{cases}$$
$$\varepsilon_2\left(\tilde{A} = (a,b,c)\right) = \begin{cases} 1 & if\ \tilde{A} \leq 0, that is, a < 0, \\ 0 & otherwise \end{cases} \tag{13}$$
$$\varepsilon_3\left(\tilde{A} = (a,b,c)\right) = \begin{cases} 1 & if\ \tilde{A} \approx 0, that is, a < 0 < c, \\ 0 & otherwise \end{cases}$$

Finally, we solve the LPP and put the above values in $\tilde{x} = (x'_j - x''_j, y_j, z'_j - z''_j) \forall j = 1,2,..., n$ to find the solution of FFLS. The solution would be termed as feasible strong fuzzy solution if $y_j - x'_j + x''_j \geq 0$ and $z'_j - z''_j - y_j \geq 0$. Otherwise the solution would be termed as infeasible weak fuzzy solution.

5.0 RESULTS AND DISCUSSION

The m x n FFLS problems of equation is solved by using decomposition and extended decomposition method. This study focuses on the comparison between decomposition and extended decomposition methods. Three examples of FFLS problem have been chosen in this study. It is interesting that the solution obtained in Example 1 and Example 2 usually same for both methods and can be solved successfully meanwhile Example 3 only can be solved by extended decomposition method.

Table 1. The comparison of Example 1 for decomposition method and extended decomposition method.

	i	**Decomposition method**	**Extended decomposition method**
x_i	1	4	4.0000
	2	1	1.0000
	3	3	2.9429
y_i	1	5	5.0000
	2	1/2	0.5227
	3	2	2.0146
z_i	1	3	3.0000
	2	1/2	0.5009
	3	1	1.0000

From table 1, we notice that the value of $\tilde{x}, \tilde{y}, \tilde{z}$, between these two types of methods quite the same. Therefore, we can consider that these two types of methods which are decomposition and extended decomposition method can be used to solve FFLS problems successfully.

Table 2. The comparison of Example 2 for decomposition method and extended decomposition method.

	i	**Decomposition method**	**Extended decomposition method**
x_i	1	36.9999	37.0000
	2	7.0000	6.2667
	3	13.3015	13.8546
y_i	1	61.9999	62.0000
	2	5.5000	5.5000
	3	4.5793	5.0000
z_i	1	74.9999	75.0000
	2	10.1999	10.8667
	3	13.9195	13.1818

From table 2, we notice that the result that was obtained by extended decomposition method quite the same as the results from decomposition method. Therefore, we also can consider that these two methods which are decomposition and extended decomposition method can be used to solve FFLS problems that not contain constraints and restriction. Moreover, these two methods can be solved FFLS problems successfully.

Table 3. The comparison of Example 3 for decomposition method and extended decomposition method.

	i	**Decomposition method**	**Extended decomposition method**
x_i	1	-	1.375
	2	-	2
	3	-	2.875
y_i	1	-	0.875
	2	-	1
	3	-	1.375

From table 3, we notice that the results for Example 3 just only be solved by using an extended decomposition method. These types of square FFLS with restriction of the crisp coefficient matrix and unrestricted fuzzy variables cannot be solved by decomposition method due to restrictions imposed by the methods. Hence, we can consider that extended decomposition method performs better than a decomposition method.

6.0 CONCLUSION

This study concentrates on applying decomposition and extended decomposition method for solving the FFLS problems. Thus FFLS are applicable in all application, and it is important to know the suitable method to be able to solve them quickly and accurately. There are many references such as in [4, 5, 6] that discuss the methods of solving real world applications which involved FFLS problems. In this research, the comparison between two methods which is decomposition and extended decomposition methods are carried out. The numerical solution of FFLS is calculated by using MAPLE programming, MATLAB programming, and LINGO solver to make computational faster.

This study also discusses about simplex and the Doolittle LU methods. At the same time, simplex and the Doolittle LU method are useful to make the computation faster and to get the final solution in these FFLS problems. Decomposition method is applicable to solve FFLS problems that not contain constraint. Decomposition method is the modified of LU decomposition method. Moreover, this decomposition method can solved the problems easier and less time compared to the LU decomposition method. Extended decomposition method has been studied to overcome decomposition method problem. This extended decomposition method is applicable to solve m x n FFLS problem of equation that contain constraint and can solve FFLS with arbitrary coefficients as well as FFLS with nonnegative and several types of restrictions.

In this research, the solutions of three examples have been investigated by both decomposition and extended decomposition methods. In the Example 1 and Example 2 of both methods, we notice that these two methods which are decomposition and extended decomposition method can be used to solve FFLS problems successfully. Moreover, the example 3 has been solved by extended decomposition method which cannot be solved by decomposition method due to restrictions imposed by the method. The results show that extended decomposition method performs better than decomposition method.

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